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ORBITAL PARAMETERS AND LOCALIZATION OF "METEOSAT"

F. Nouel

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SUMMARY

PURPOSE OF THE REPORT

We have examined here two problems concerning the METEOSAT project. The present report is thus divided into two relatively distinct parts:

Chapter I.

Study of nominal values of orbtial parameters as functions of the requirements for restoration of the image supplied by the satellite.

Chapter II.

Study of methods for localization, and the influence of various sources of error on restoration of the geographic position of the satellite.

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ABSTRACT: The factors influencing the trajectory control of the 24 hour French Earth satellite METEOSAT are described. Computer simulation of the drift in position and orbit determination were made. It was found that the longitude is more poorly determined than either the latitude or radial distance.

CHAPTER I /2*

ORBITAL PARAMETERS

I. Introduction

This is a reply to note LB/GD/0.307/CB/MT of 16 June 1970, which raises a number of problems about the orbit of METEOSAT. This report is based principally on a CNES report by J. C. Blaive (JCB/GD/9.157), titled, "Maintenance of the Position of a 24-hour Equitorial Satellite".

II. Outline and Consideration of the Problem

1.1. Restoration of the image supplied by a satellite involves constraints on the limits of the displacement of the satellite during a time Δt; it must remain in a cube* P determined by a limit in longitude $\Delta\lambda$, a limit in latitude $\Delta\mu$, and a limit in geocentricity Δr.

* Numbers in the margin indicate the pagination of the original foreign text.

* Translators Note: Literally "paving stone".

Several cubes must be considered, depending on the mode of restoration chosen.

We consider the following cubes P, and their corresponding limits:

P_1	$\Delta\lambda$	0.360 km
	$\Delta\mu$	0. 360 km
	Δr	2. 750 km
	Δt	25 minutes
P_2	$\Delta\lambda$	0. 360 km
	$\Delta\mu$	0.360 km
	Δr	3. 14 km
	Δt	20 minutes
P_3	$\Delta\lambda$	2.9 km
	$\Delta\mu$	2.8 km
	Δr	30 km
	Δt	2 h 30 mn
P_4	$\Delta\lambda$	5.3 km
	$\Delta\mu$	5.3 km
	Δr	30 km
	Δt	2 h 30 mn

1.2. The apparent movement of the satellite is the superposition of:

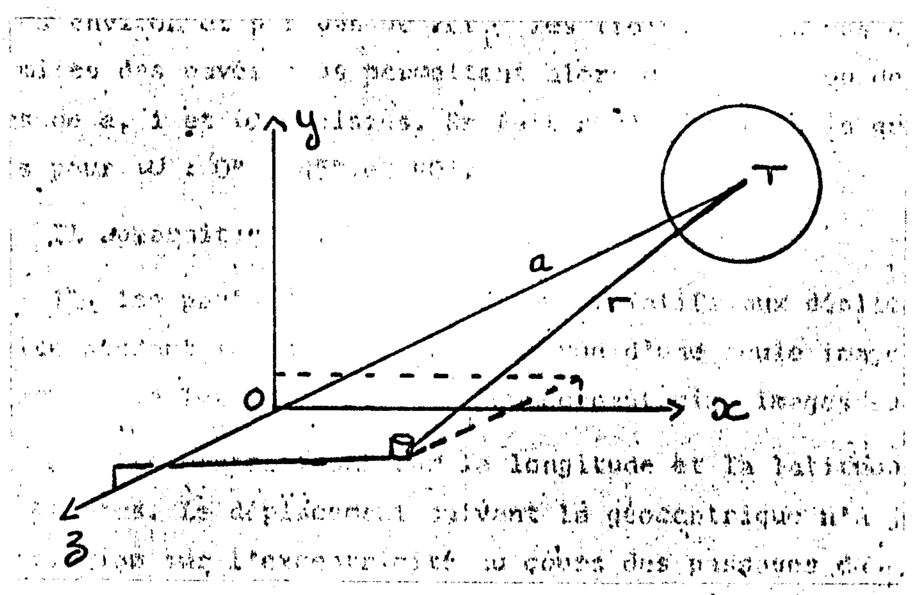
1) diurnal movement about a mean point;
2) drifting of this point.

The mean point is defined as being the position which the satellite would occupy if the inclination and the eccentricity were zero, and if the period were rigorously 24 hours. /3

a) Diurnal Motion

To describe the diurnal movement, we shall use the following coordinate system Ox y z:

O is the mean point of the satellite;
Ox is tangent to the orbit;
Oy is orthogonal to the orbital plane;
Oz passes through the center of the Earth, T.



The coordinates of the satellite in this system are given by: /4

$$x \simeq 2 r e \sin nt$$
$$y \simeq r i \sin (\omega + nt)$$
$$z \simeq a (1 - e \cos E)$$

where

- r is the radius vector from the center of the Earth to the satellite;
- n is the mean motion;
- w is the argument of the perigee;
- a is the semimajor axis -- in fact, the radius of the circular orbit of the mean point;
- E is the eccentric anomaly given by Kepler's equation $E - e \sin E = nt$.

As a consequence:

- x represents motion in longitude;
- y represents motion in latitude;
- z represents geocentric motion.

The times Δt_i lie between several seconds and several hours, so it can be seen that the limits on the displacements must be considered: first of all, the diurnal motion.

Taking $a = 42\ 160$ km, we have tabulated the three coordinates of the satellite over a period of eight hours, in steps of Δt_i. The cube limits allow us then to accept or reject the values of e, i, and w chosen. As a matter of fact, we have taken only three values for w: 0°, 45°, and 90°.

It can be seen that:

1. Cubes P_1 and P_2, which refer to displacements of the satellite while a single image is being taken, are more restrictive than cubes P_3 and P_4, which relate to five successive images.

2. Restrictions in longitude and latitude are preponderant. Geocentric displacement has never imposed any restriction on the eccentricity during the runs of this program.

Eccentricity is conditioned by the longitude, while the latitude affects the inclination and the argument of the perigee, as could be expected.

3. The form of the ellipse in the Oxy plane depends on w. For w = 45°, the limit on i is the least restrictive. But in reality, the argument of the perigee w will vary continuously, so that the limits are chosen as functions of the most unfavorable value of w. /5

4. Cubes P_1 and P_2 require an eccentricity of the order of 3×10^{-5} and 4×10^{-5} and an inclination of the order of 6×10^{-3} to 8×10^{-3} degree. Such values have led us to take either cubes 10 P_i with a time Δt_i, or cubes P_i with times 10^{-1} Δt_i or 10^{-2} Δt_i.

Furthermore, comparison among the first results obtained seems to show that the requirements on e and i are proportional to multiples of the dimensions of the cubes P_i, or inversely proportional to multiples of the times Δt_i.

5. Principal results.

TABLE 1

CUBE		UPPER LIMIT TO e	UPPER LIMIT TO i IN DEGREES
P_1	Δt_1	3.10^{-5}	4.10^{-3}
P_3	Δt_3	5.10^{-5}	6.10^{-3}
P_2	Δt_2	$0.4.10^{-4}$	$0.5.10^{-2}$
P_4	Δt_4	1.10^{-4}	$1.1.10^{-2}$
$2P_1$	Δt_1	$0.7.10^{-4}$	$0.8.10^{-2}$
$2P_3$	Δt_3	$1.1.10^{-4}$	$1.2.10^{-2}$
$10P_1$	Δt_1	3.10^{-4}	4.10^{-2}
$10P_3$	Δt_3	5.10^{-4}	6.10^{-2}

TABLE 1. Cont.

CUBE		UPPER LIMIT TO e	UPPER LIMIT TO i IN DEGREES
P_1	$10^{-1}\Delta t_1$	3.10^{-4}	4.10^{-2}
P_3	$10^{-1}\Delta t_3$	5.10^{-4}	5.10^{-2}
P_1	$10^{-2}\Delta t_1$	3.10^{-3}	0.4
P_3	$10^{-2}\Delta t_3$	5.10^{-3}	0.5

In this table, the limits to e and i are determined by starting with a displacement in longitude $d\lambda$ and a displacement in latitude $d\mu$. In the definition of the cubes P_i in the note cited in reference, the bounds of the cubes P_i are $\pm\, d\lambda_i$ and $\pm\, d\lambda_i$; it is thus necessary to multiply the results by two. This has been taken into consideration in the conclusion. /6

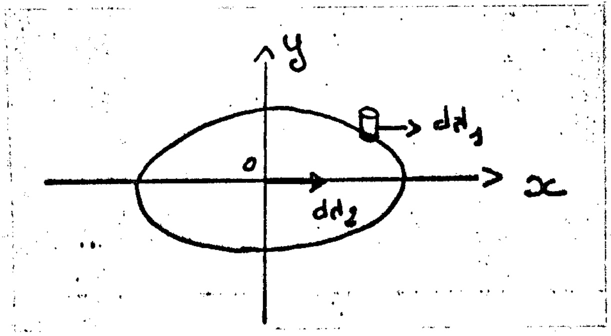
b) Drift of the Mean Point.

Perturbing effects on the moon and sun, anomalies in terrestrial potential, and radiation pressure cause drift in the motion of the mean point. As the terrestrial potential predominates in the longitude, we shall study this case somewhat later.

Table 2 shows the effects of the other two causes.

It can be determined that the diurnal drifting of the orbital parameters does not impose any restrictions on these. It is not the same for the drift in longitude due to the terrestrial potential, about which we shall now speak.

The apparent movement of the mean point depends on the difference Δn between the mean motion of the satellite and the rotation of the Earth. If $d\lambda_1$ represents the displacement of the satellite in its diurnal motion about the mean point, and if $d\lambda_2$ is the displacement of this latter, there results a variation of $d\lambda_1 + d\lambda_2$.



Studies have shown that $d\lambda_2$ can be distinctly larger than the limit $d\lambda$ fixed by a cube. We must thus introduce a restriction on Δn, since it does not seem reasonable to carry out trajectory corrections during the time a picture is being taken.

In the note cited above, there are curves giving the drift in longitude as a function of longitude for constant Δn.

As an example, we have taken the conditions $\left\{10P_1\ ,\ \Delta t_1\right\}$. The limit /8
$d\lambda$ requires a drift less than 0.275 degrees per day, which yields the following (Δn expressed in degrees per day, and $da = -\frac{2}{3}\frac{a}{n}\,dn$):

For	$\lambda = 0°$:	$0.42 < \Delta n < 0.5$	soit	33,6 km < or < 40 km
	$\lambda = 10°$:	$0.40 < \Delta n < 0.5$	soit	32 km < or < 40 km
	$\lambda = 20°$:	$0.34 < \Delta n < 044$	soit	27 km < or < 35 km

It is seen that the accuracy of the semimajor axis da is a function of the longitude λ. The curves do not allow results to be given for all cases imaginable, so we have made an approximate calculation, which nevertheless gives a representative order of magnitude.

For the cube

P_1	$da \simeq 2.2$ km
P_2	$da \simeq 2.7$ km
P_3	$da \simeq 2.9$ km
P_4	$da \simeq 5.4$ km

TABLE 2

Perturbation	On i	On e	On λ
Lunar - Solar	Secular 2.6×10^{-3} °/day Periodic Negligible	Secular 2×10^{-2} e/year Periodic Moon: amplitude 10^{-2}e period 14 days Sun: amplitude 1.3×10^{-2}e period 180 days	Secular 7×10^{-3} °/day Periodic Moon: amplitude 6.6×10^{-5} period 14 days (i.e., 0.9×10^{-5} °/day) Sun: amplitude 6.6×10^{-4} period 180 days (i.e., 3.7×10^{-6} °/day)
Radiation pressure	1.15×10^{-4} e °/day	amplitude 4×10^{-4} period 1 year	Secular 3×10^{-7} radian/year (i.e., 1.72×10^{-5} °/day Periodic Amplitude 2×10^{-2}e Period 1 year

III. Simulation of the Motion

1. A first simulation was made over a period of one and one-half days. The orbital parameters were:

$$a = 42\ 162\ \text{km}$$
$$e = 0.2\ 10^{-3}$$
$$i = 0.4\ 10^{-1}$$
$$\omega = 45^\circ \qquad \text{with the coefficient } S/M = 0.235 \text{ m/kg}$$
$$\Omega = 0$$
$$M = 0$$

Curves showing the variations of a, e, and i have been traced for this time period (Figures 1, 2, and 3).

It can be seen, for example, that the parameters remain within the limits fixed by the conditions $\left\{10P_1, \Delta t_1\right\}$.

2. With identical parameters, except for $\Omega = 218.497^\circ$ and $M = 315^\circ$, we have simulated the motion of the satellite during a period of one month. In this case S/M = 0.1 m /kg.

With these parameters, the apparent motion is not the ellipse assumed in the preceding calculations. However, it has been determined that drifts in e and i during a period of at least 16 days do not fall outside the region allowed by the condition $\left\{10P_1, \Delta t_1\right\}$. /9

Figures 4 and 5 show the variations of e and of i, respectively, with 15 days elapsed between the two curves of each Figure. Figures 6 and 7 show the variations of e, i, and a during this 15-day period. Figure 8 describes the apparent motion with the chosen orbital parameters.

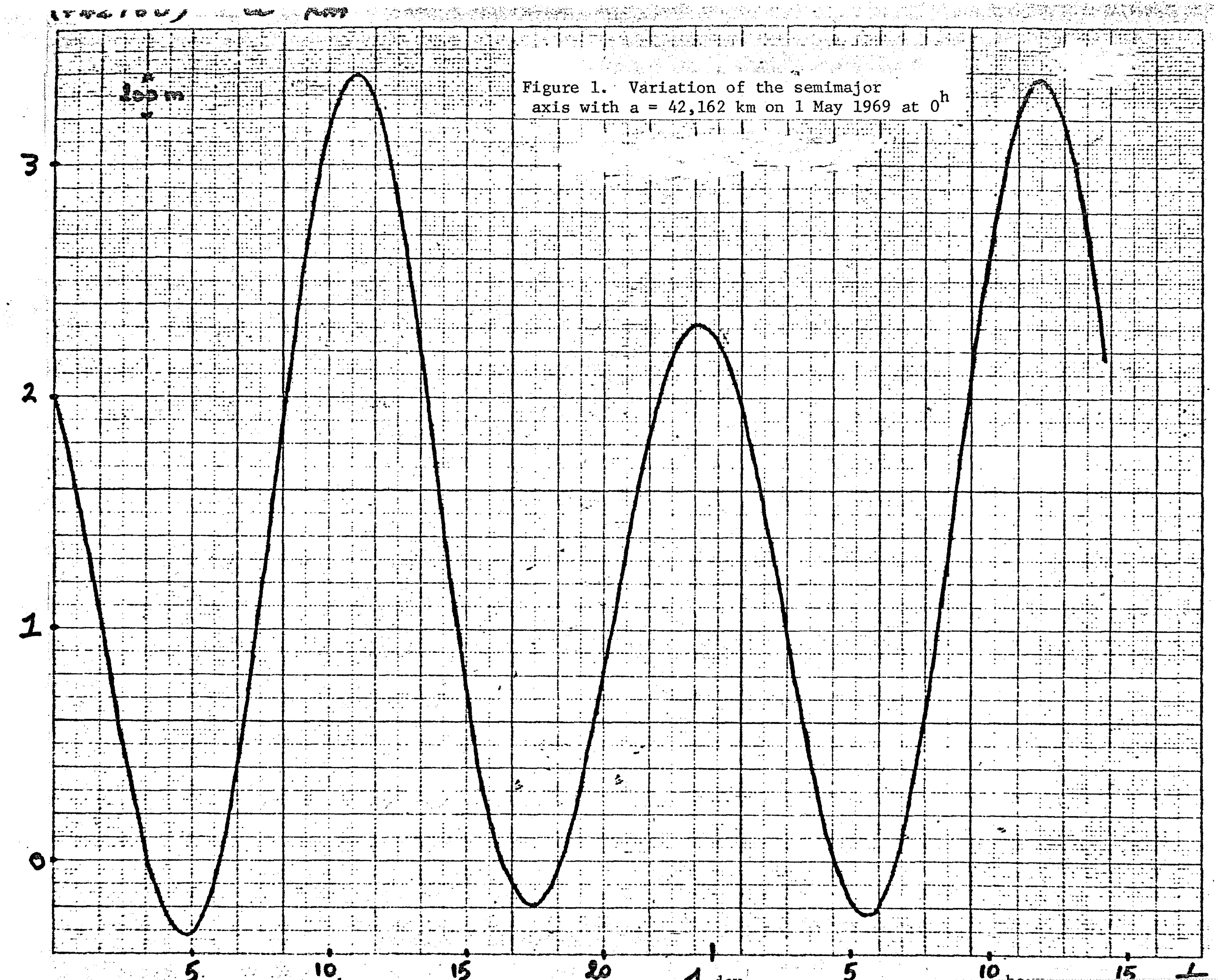


Figure 1. Variation of the semimajor axis with a = 42,162 km on 1 May 1969 at 0^h

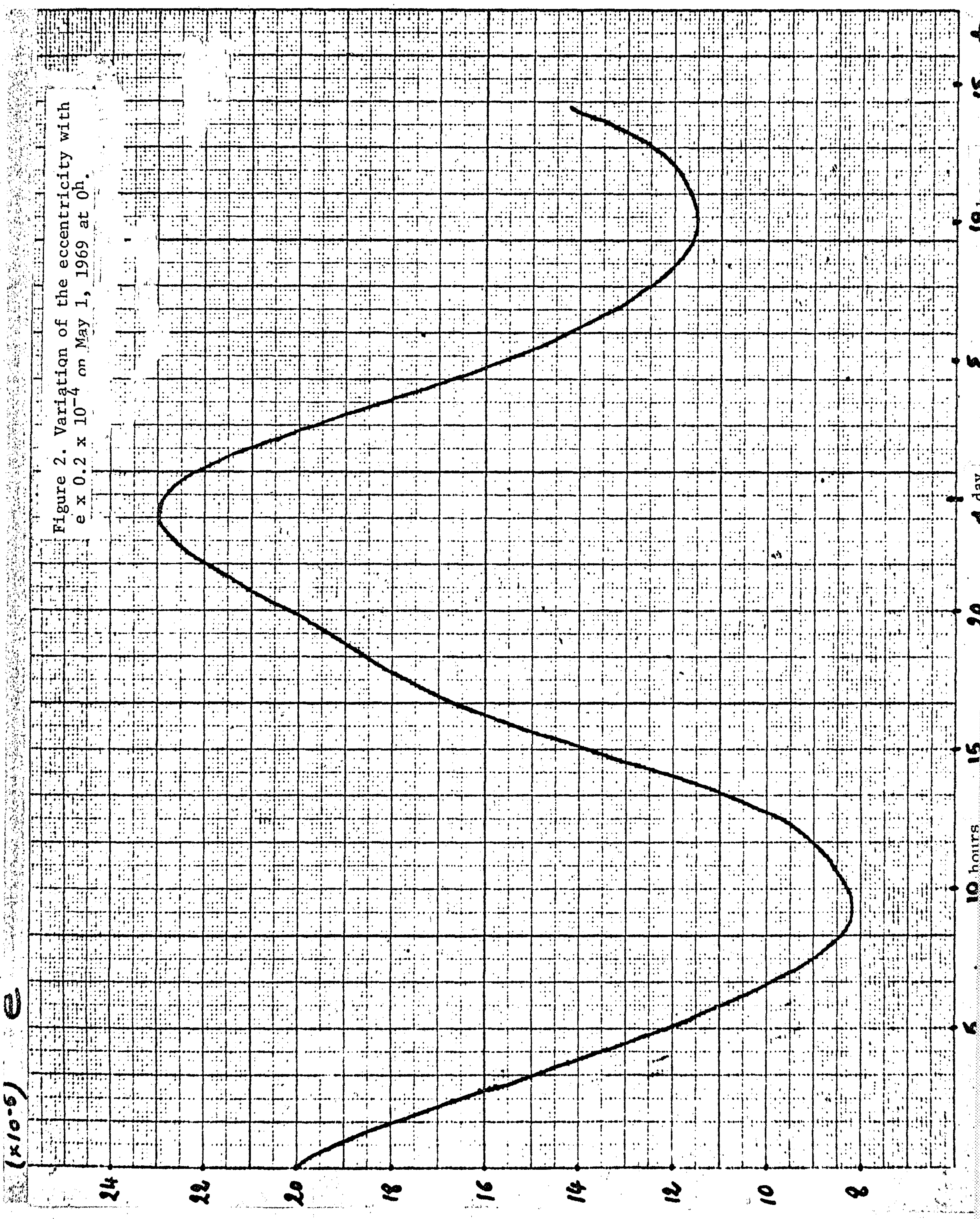


Figure 2. Variation of the eccentricity with e x 0.2 x 10^{-4} on May 1, 1969 at 0^h.

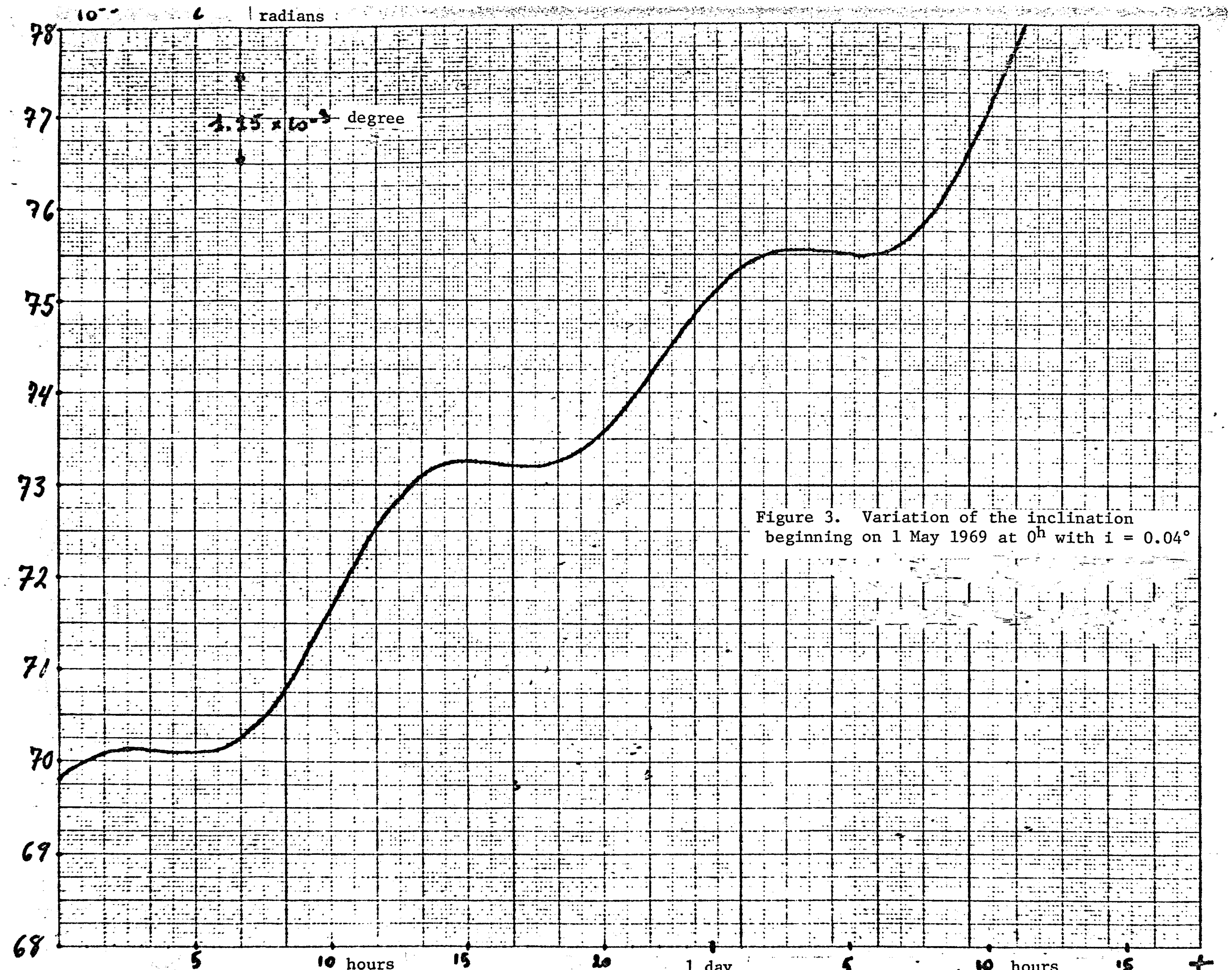


Figure 3. Variation of the inclination beginning on 1 May 1969 at 0^h with $i = 0.04°$

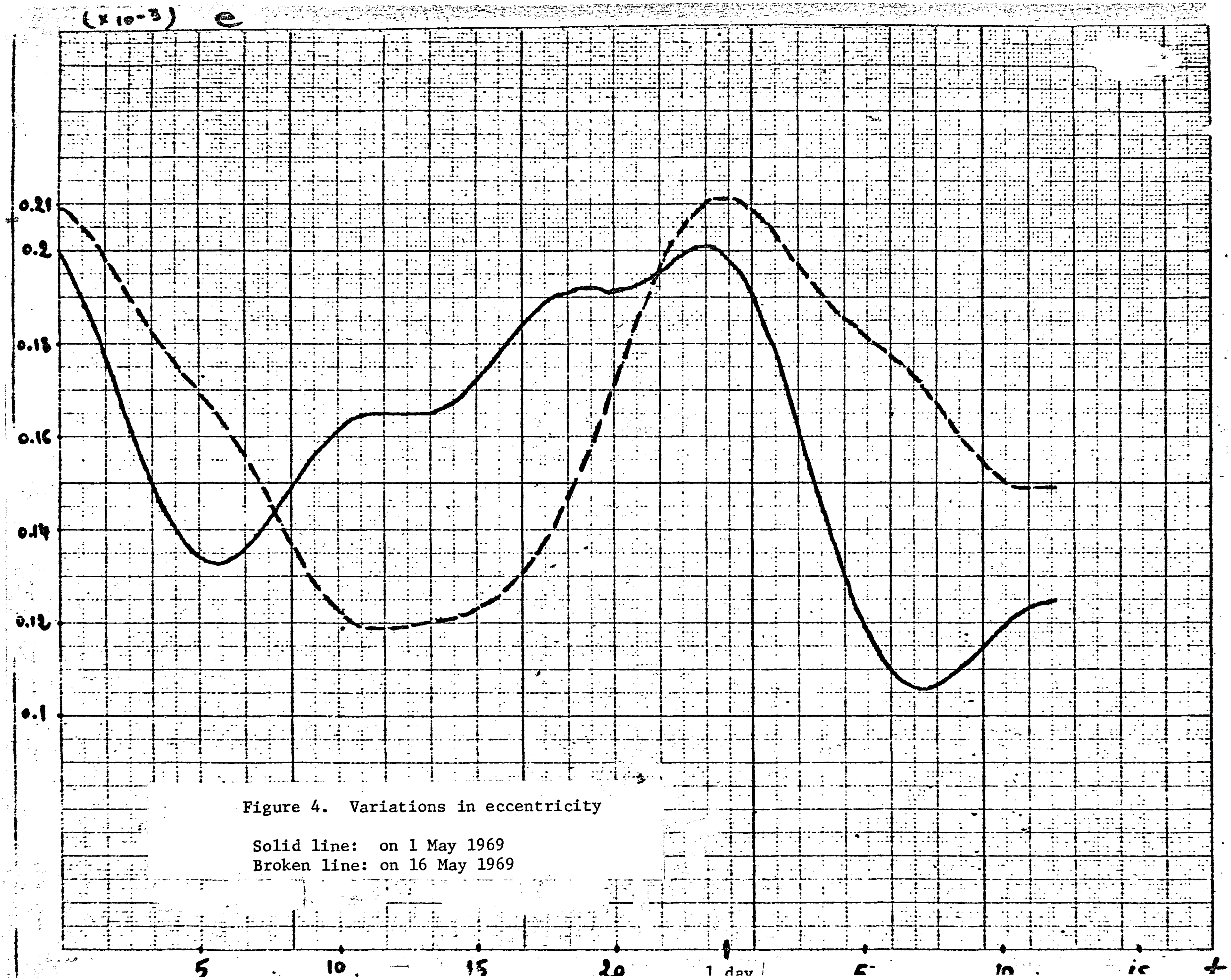


Figure 4. Variations in eccentricity

Solid line: on 1 May 1969
Broken line: on 16 May 1969

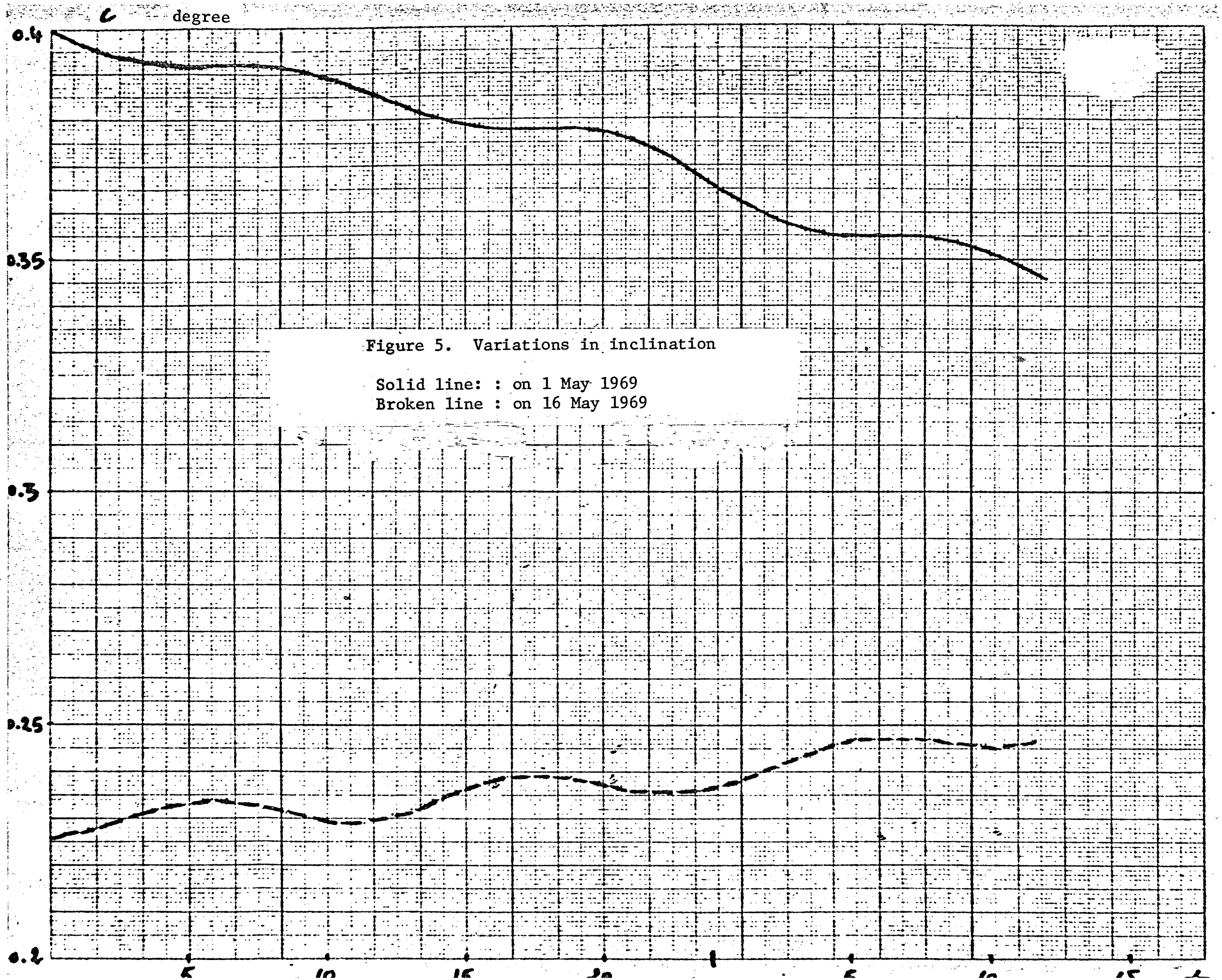


Figure 5. Variations in inclination

Solid line: : on 1 May 1969
Broken line : on 16 May 1969

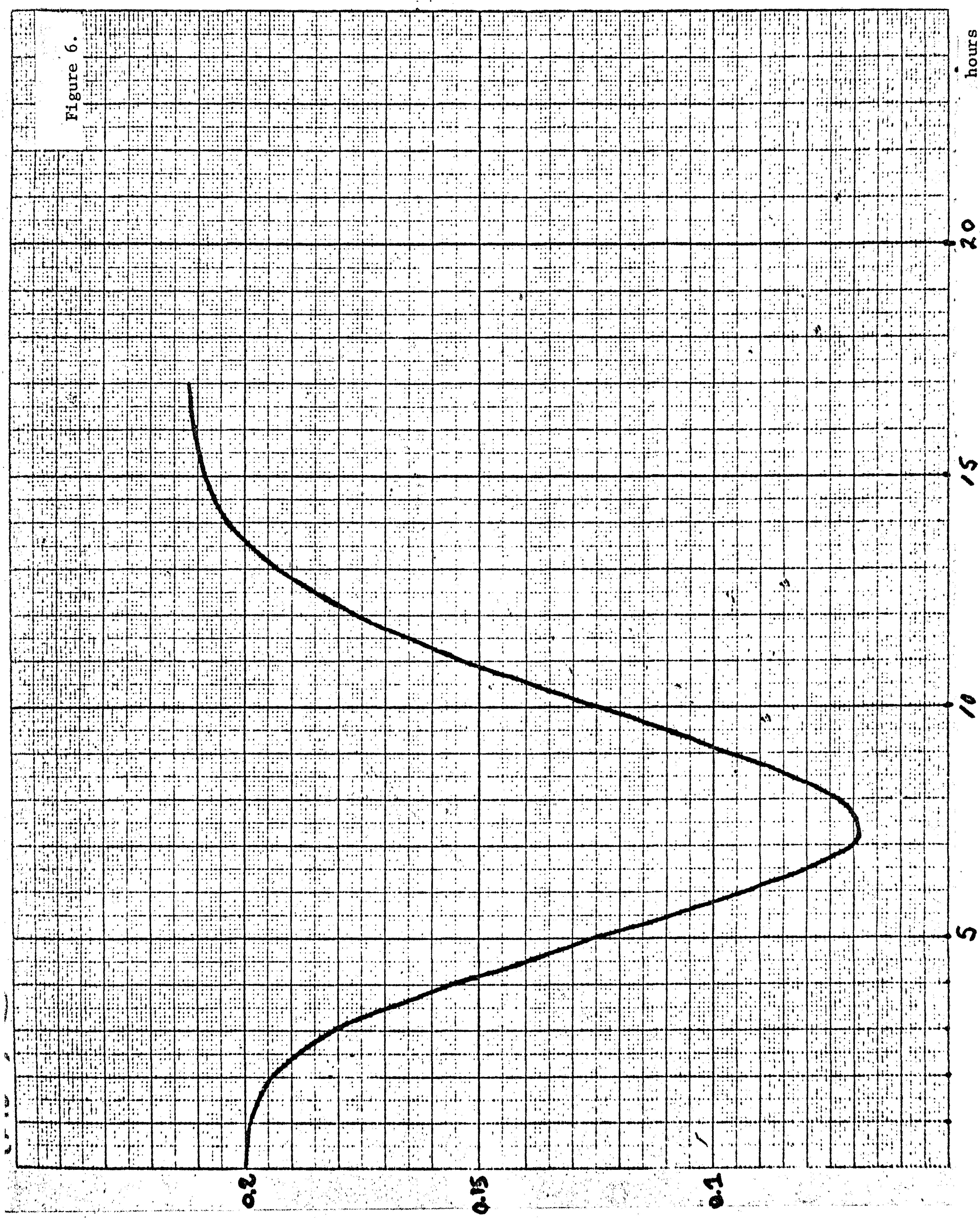


Figure 6.

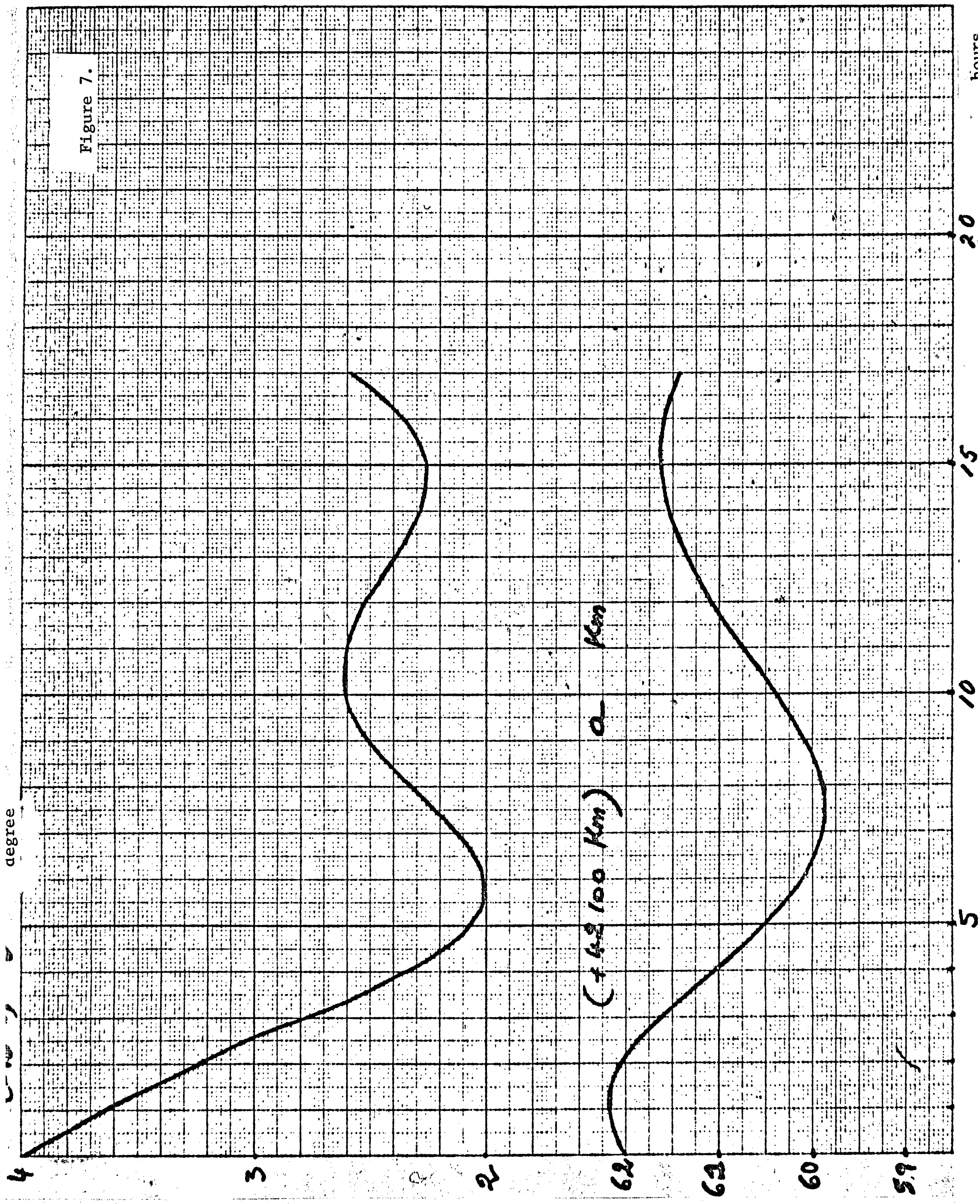


Figure 7.

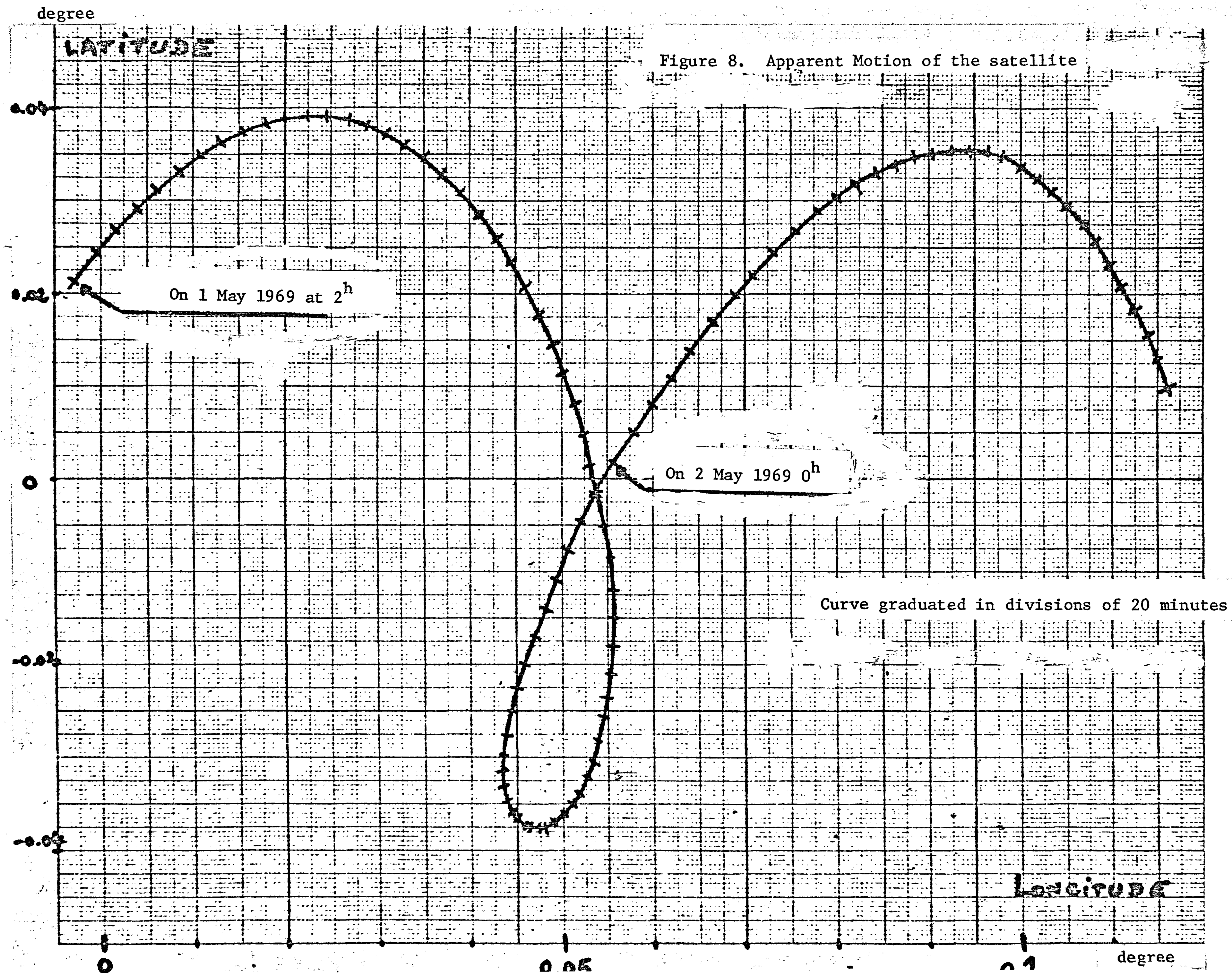


Figure 8. Apparent Motion of the satellite

IV. Conclusion

A number of results are collected in Table 3:

TABLE 3

CONDITIONS	DISPLACEMENT IN LONGITUDE		DISPLACEMENT IN LATITUDE	
	Limit on e	Order of magmitude of the accuracy of the semimajor axis km	Limit on i, in degree	Approximate annual number of corrections
$P_1 \quad \Delta t_1$	3.10^{-5}	2.2	0.008	70
$P_2 \quad \Delta t_2$	4.10^{-5}	2.7	0.01	50
$10P_1 \quad \Delta t_1$	3.10^{-4}	30	0.08	7

If one wishes an estimate of the limits on e and i for a cube $\{nP_i ; \Delta t_i\}$ or $\{P_i , k \Delta t_i\}$, it is sufficient to multiply the limits for the cube $\{P_i , \Delta t_i\}$ by n or 1/k.

The corrections in longitude are not functions of the requirements for image restoration, but rather of the geographic position of the satellite and of the tolerance with respect to this stationing point.

From the experience of the Americans in this area, it seems to be difficult to maintain an eccentricity less than 10^{-4}. /10

Corrections in inclination are the most costly, and require expenditures of the order of 50 m/sec. per year in every case; only the annual number of these corrections varies.

CHAPTER II

LOCALIZATION

I. Introduction

In this chapter we wish to study methods for localizing a stationary satellite, and the accuracy of restoration of its position. /11

We have used a simplified force model to simulate the data and to process them. In fact, it is not necessary to use a complete model to see the influence of the different parameters, unless of course, one wishes to examine the errors due to the model itself.

We have thus proceeded by comparing the restored position based on a group of "perfect" measurements, with that obtained from a group of simulated measurements containing a source of error. The accuracy is characterized by deviations in longitude $|\Delta\phi|$, in latitude $|\Delta\lambda|$, and in altitude Δh, from the perfect trajectory.

Among the various factors affecting the accuracy, we have examined the influence:

- of the accuracy of knowledge of the position of the stations,
- of noise and of biases in the measurements.

We are working with measurements of distance and with angular measurements. In restoring the position, it is also necessary to take into account the geometric position of the stations, and the frequency of the observations.

It was not possible to consider all configurations, and to tabulate the influence of each parameter in every case. The data-processing method is now available, and can be used each time it will be necessary to study a particular case.

Nevertheless, we can draw some preliminary conclusions from the results already obtained. They are given for random and for systematic errors. It can be affirmed that for different numerical values, the results can be derived by a simple multiplicative coefficient.

II. Method for Estimating the Accuracy of Localization /12

Simulation of the data was done by means of the program SIDOLA. The model of the terrestrial potential included only the first terms of the expansion (out to J_5). It is in fact unnecessary to take a developed force model (Standard-Earth, lunar-solar effects, radiation pressure) to consider the influence of various sources of error in the restoration of the trajectory.

A first group of data was thus created, and constitutes the "perfect" measurements. It was processed with a differential correction program (LIDO) which gave statements of the orbital parameters.

These statements were then used in the POSAT program, which gives longitudes, latitudes, and altitudes of the satellite at regular intervals of time. These several parameters will be called the "nominal trajectory of the satellite".

The same sequence of processing was used for:

1) measurements containing noise introduced by a subroutine which generated pseudo-random numbers;
2) measurements containing both noise and a systematic error;
3) measurements obtained by displacing the station.

In each of these cases, the statements allowed establishment of a trajectory which, compared to the nominal trajectory, showed the effect of the source of error on the restoration of the position of the satellites.

III. Presentation of Results

1. Choice of numerical values

- The orbital parameters were chosen to place the satellite near the point with geographic coordinates:

$$\varphi = 0^\circ \quad , \qquad \lambda = 0^\circ \quad , \qquad h = 35\,786 \text{ km}$$

- The stations, which are KOUROU (KRU) and HAUTE-PROVENCE (HPL), were displaced 50 meters in longitude and latitude, and 20 meters in altitude:

KRU	$\varphi = 5^\circ.25$	HPL	$\varphi = 43^\circ.932$
	$\lambda = 52^\circ.805$		$\lambda = 354^\circ.287$
	h = 10 mètres		h = 0.763 km

- In the case of a distance measurement, the noise and the systematic error are 100 m. As for angular measurements, they are the cosines of angles which have noise or bias of 2×10^{-4}.

2. Pattern of measurement

There are three sorts of simulation relating to the distribution of the measurements in time.

Case A A group of measurements at intervals of one minute, and lasting for a quarter-hour. We make use of one such group every four hours for a period of four days.

Case B A group of measurements at intervals of one minute, and lasting for a quarter-hour. We make use of one such group every 12 hours for a period of 5 days.

Case C A group of measurements at intervals of two minutes, and lasting for a half-hour. We make use of one such group every 24 hours for a period of 8 days.

- Extrapolation will be made for: 1 day in Case A
  2 days in Case B
  3 days in Case C

3. Results

(see Table 4)

The results are presented in this table, with the various cases indicated by a numeral, as follows:

1 - measurement of distance from HPL, simulation A
2 - measurement of distance from HPL, simulation B
3 - measurement of distance from HPL, simulation C
4 - angular measurements from HPL, simulation A /14
5 - angular measurements from HPL, simulation B
6 - angular measurements plus distance measurements from HPL, simulation B
7 - angular measurements from KRU, simulation A
8 - angular measurements from HPL, simulation A. This case is analogous to No. 4, but the processing by differential correction is different, and resembles Case No. 7.
9 - angular measurements from HPL and KRU, simulation A

In the same table, we denote by:

Noise: deviations which introduce noise into the measurements (random errors).

Noise + bias: deviations due to the presence of noise and of a systematic error.

Stations Displaced: deviations produced by displacement of the station during simulation.

In each Case studied, there are three numbers, which represent successively:

- deviation in latitude $\Delta\phi$
- deviation in longitude $\Delta\lambda$
- deviation in altitude Δh

$\Delta\phi$ and $\Delta\lambda$ are expressed in radians.
$\Delta\phi$, $\Delta\lambda$ and Δh are the maximal deviations in one day if the simulation is of type A, and in two days if it is of type B.

Figures 9, 10, 11, and 12 show that these deviations have periodic behaviour.

IV. Analysis of Various Factors Affecting Accuracy of Localization

1. Geometrical coverage

a) Geographic position of the station

Cases 7 and 8 are identical with respect to treatment: they refer to angular measurements from stations KRU (Case 7) and HPL (Case 8).

Table 4.

Case	Extrapolation		Noise		Noise + Bias		Stations displaced	
1	$\Delta\varphi$: < 0.87 10^{-7}		1.46 10^{-5}	613 m	1.46 10^{-5}	613 m	< 0.87 10^{-7}	
	$\Delta\lambda$: 1.92 10^{-5}	806 m	2.44 10^{-5}	1.025 km	11.87 10^{-5}	4.985 km	6.56 10^{-5}	2.755 km
	Δh : 10 m		82 m		80 m		9 m	
2	0.14 10^{-5}	59 m			19.10^{-5}	7.98 km	1.15 10^{-5}	483 m
	0.07 10^{-5}	29 m			11.5 10^{-5}	4.830 km	5.10^{-5}	2.100 km
	9 m				890 m		57 m	
3	0.35 10^{-5}	147 m	3.09 10^{-5}	1.298 km	7.89 10^{-5}	3.314 km	0.44 10^{-5}	185 m
	3.56 10^{-5}	1.495 km	18.6 10^{-5}	7.812 km	76.8 10^{-5}	32.256 km	0.91 10^{-5}	382 m
	26 m		149 m		613 m		32 m	
4	< 0.87 10^{-7}		1.5 10^{-5}	630 m	2.2 10^{-5}	924 m	0.1 10^{-5}	42 m
	3.49 10^{-5}	1.466 km	1.66 10^{-5}	697 m	17.9 10^{-5}	7.518 km	0.94 10^{-5}	395 m
	1 m		102 m		81 m		5 m	
5	0.02 10^{-5}	8 m	4.22 10^{-5}	1.772 km	26.3 10^{-5}	11.046 km	1.2 10^{-5}	504 m
	< 0.87 10^{-7}		2.62 10^{-5}	1.1 km	17.6 10^{-5}	7.392 km	2.84 10^{-5}	1.193 km
	1.5 m		173 m		395 m		397 m	
6	0.49 10^{-5}	206 m			0.44 10^{-5}	184 m	1.1 10^{-5}	462 m
	0.21 10^{-5}	88 m			3.49 10^{-5}	1.466 km	4.97 10^{-5}	2.087 km
	23 m				54 m		55 m	
7					0.58 10^{-5}	244 m	0.05 10^{-5}	21 m
					14.08 10^{-5}	5.914 km	2.58 10^{-5}	1.084 km
					1.876 km		163 m	
8					0.68 10^{-5}	286 m	0.07 10^{-5}	29 m
					4.31 10^{-5}	1.810 km	2.56 10^{-5}	1.075 km
					792 m		162 m	
9					0.52 10^{-5}	218 m	< 0.87 10^{-7}	
					7.97 10^{-5}	3.347 km	0.35 10^{-5}	147 m
					1.043 km		45 m	

10^{-5} radians $\Leftrightarrow$ 420 m

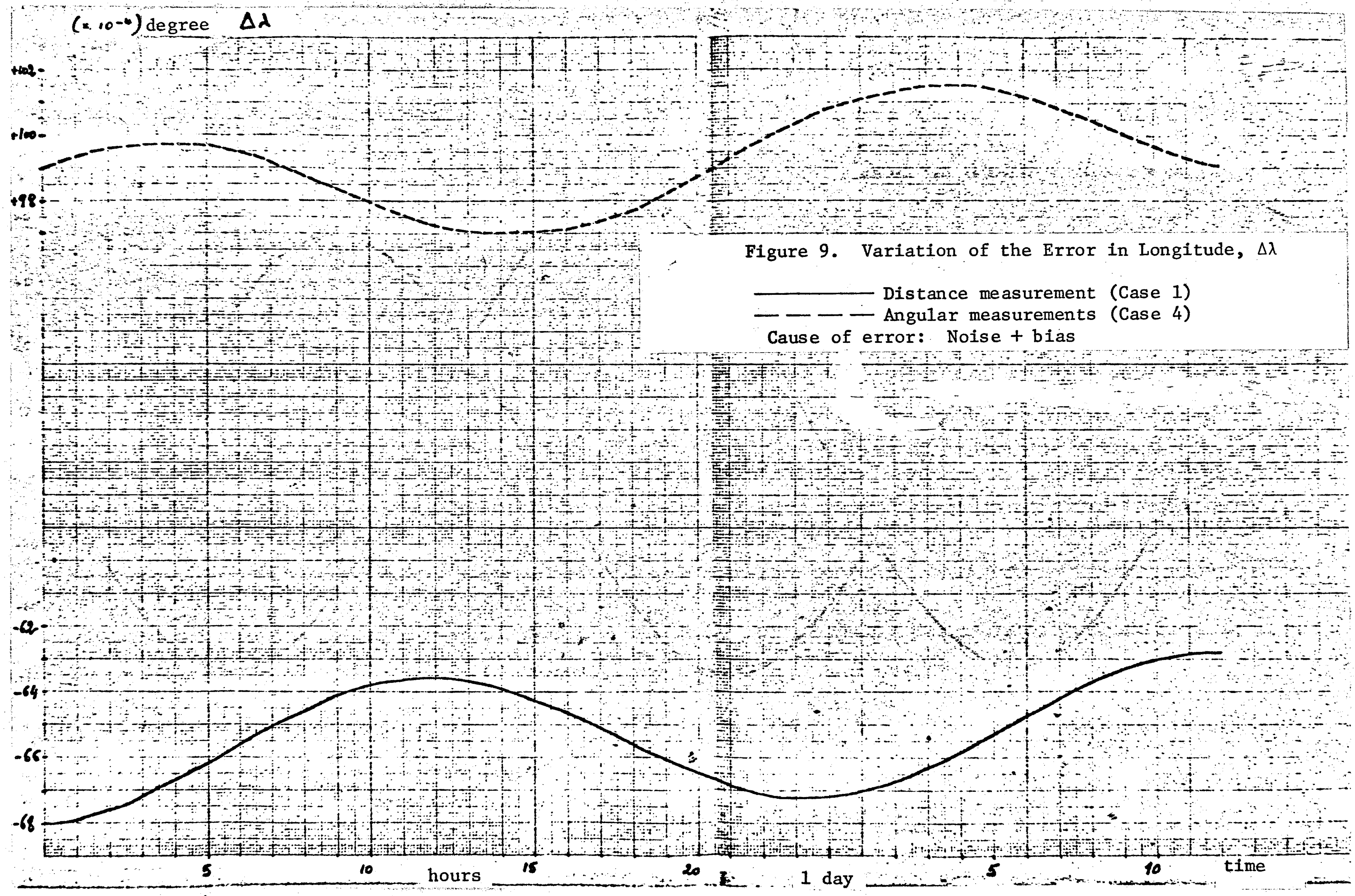


Figure 9. Variation of the Error in Longitude, Δλ

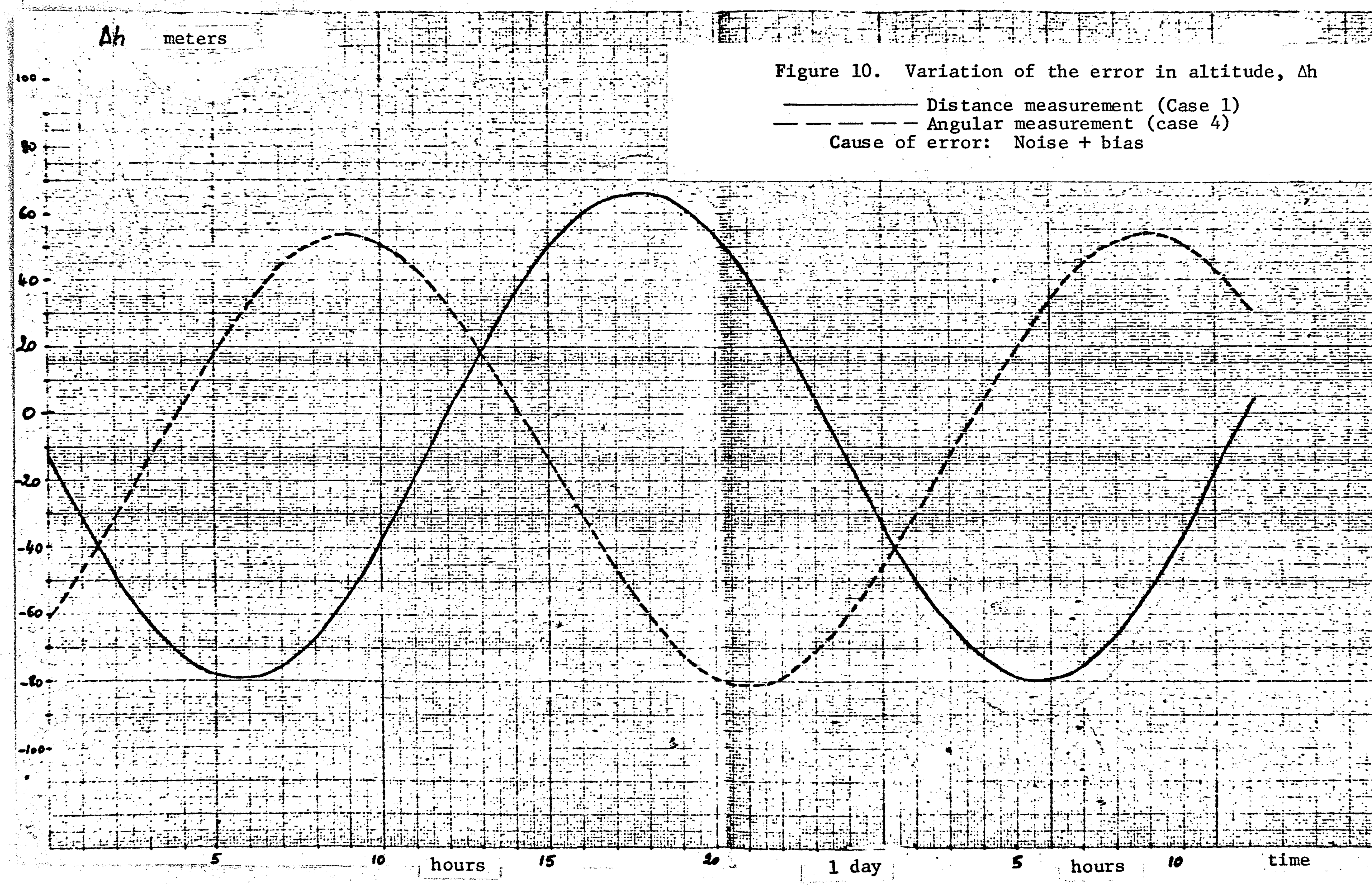


Figure 10. Variation of the error in altitude, Δh

——————— Distance measurement (Case 1)
— — — — — — Angular measurement (case 4)
Cause of error: Noise + bias

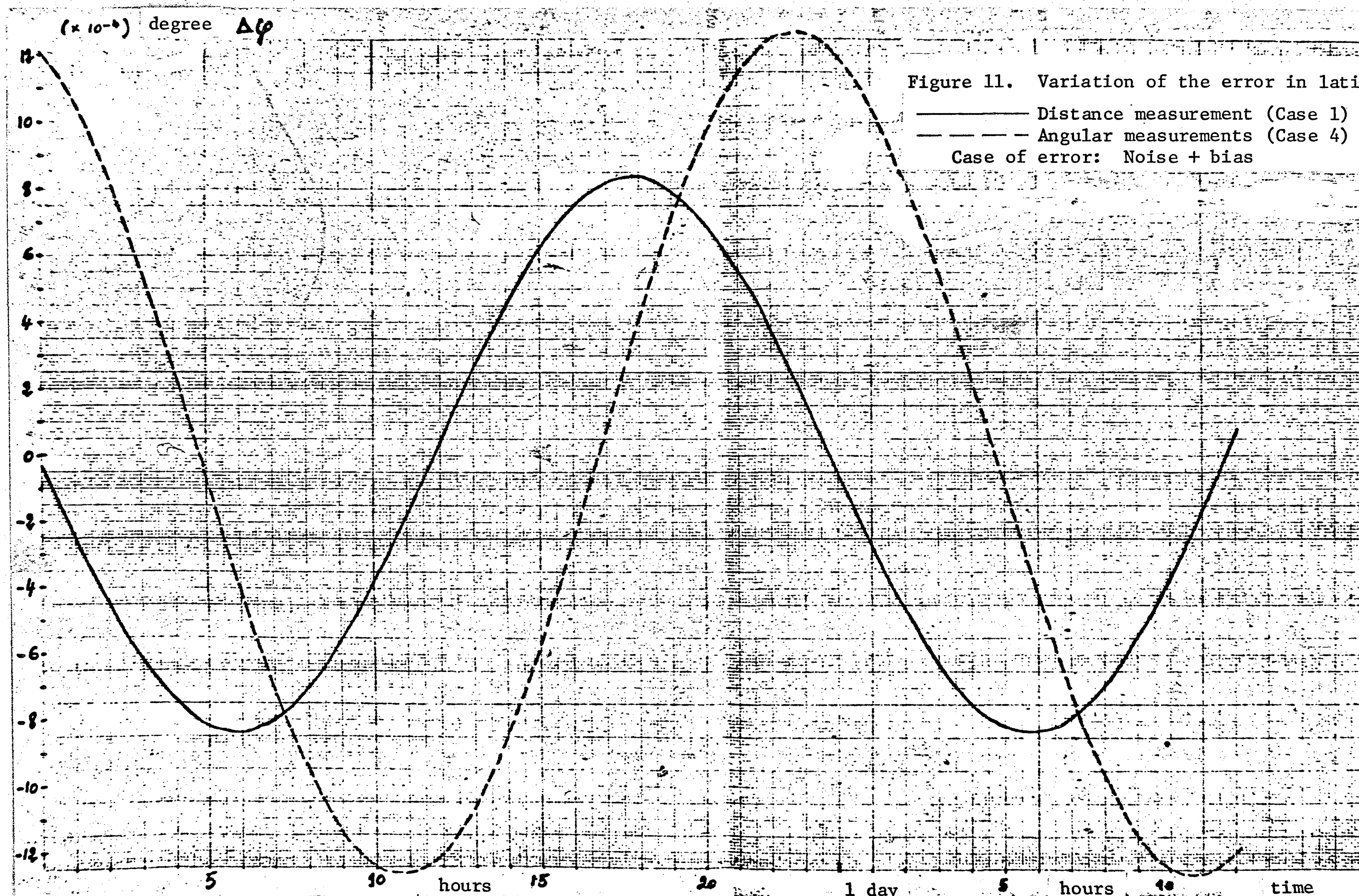


Figure 11. Variation of the error in latitude,
Distance measurement (Case 1)
Angular measurements (Case 4)
Case of error: Noise + bias

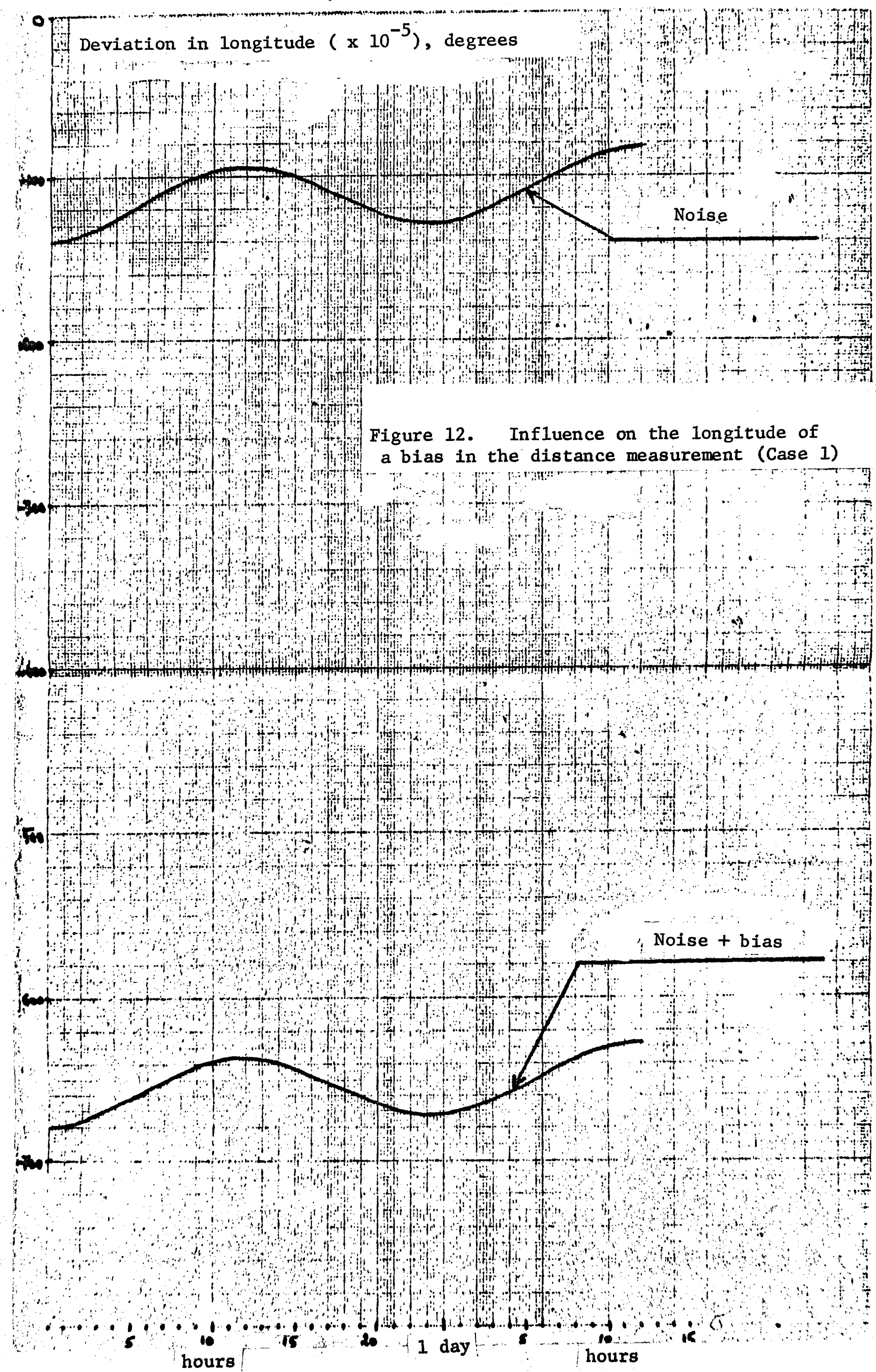


Figure 12. Influence on the longitude of a bias in the distance measurement (Case 1)

In the study of the effect of noise and bias, a factor of two was discovered in the restoration of position in longitude and latitude.

This could be interpreted by examining a cosine curve. A bias of 2×10^{-4} in a cosine does not result in the same bias for all angles. Now, the cosines of the longitudinal positions are 0.87 and 0.76 for KRU and HPL, respectively. The bias introduced in the angle is thus larger for KRU than for HPL.

b) Number of tracking stations

We have treated a case analogous to the two preceding ones, but with measurements from KRU and HPL used at the same time (Case 9). Station HPL by itself would allow better restoration. The restoration produced by use of both stations together lies between those produced by their use separately.

This fact is not surprising, since unlike the situation with a low satellite, a single station provides complete orbital coverage for a stationary satellite. However, presence of several tracking stations would reveal systematic errors caused by one of their number.

2. Force model used

A study already carried out for the A.T.S. 3 satellite (JPL/JJ/GD/9.548/CB/MT) shows that "it is the quality of the measurements and not the force model used which limits adjustment of the orbital parameters from the measurements".

The first column of the table, called "extrapolation", includes to a certain extent the effect of the potential model on the localization. By treating only part of the measurements and extrapolating over an interval of time, the orbital parameters which serve to restore the trajectory are modified, and the deviations do indeed reflect the deviations of the model. /17

3. Knowledge of the stations' positions

The error introduced by displacement of the stations is not negligible. We have simulated the measurements by displacing the stations 50 m in longitude and in latitude and 20 m in altitude, while retaining their former positions for the treatment.

For example, in Case 4, the error in longitude is of the order of 400 meters. It would be desirable to obtain improvement in knowledge of the positions of the stations by a factor of five, using other programs.

4. Type of measurements and accuracy

4.1 Two sorts of measurements have been used: distance and angular measurements.

The results obtained for distance are analogous to those obtained for angular measurements. This may appear surprising, since the errors introduced are not compatable (100 m in distance and 2×10^{-4} in cosines of the angles). In reality, the angular measurements are composed of two angles, and thus offer a greater diversity than the measurement of distance alone.

As an example, we show the cconfidence limits for parameters a, e, and i obtained in Cases 1 and 4 (Table 5).

4.2. Mixed measurements

We have attempted to simulate what would result from use of a self-directing antenna by using one distance measurement and two angular measurements from Haute-Provence (Case 6).

Considering the nominal accuracy of the measurements, the results are indeed better than for Case 2 (distance alone) or for Case 5 (angles alone).

5. Rate of observation

Cases 1, 2, and 3 are distinguished only by the observation rate, as are Cases 4 and 5. It seems essential to have good coverage of the orbit, which requires several observations each day (≥ 3) for a stationary 24-hour satellite. In cases 2, 3, and 5, the same arcs of the orbit are always observed, and the adjustment occurs only on these arcs, which results in a degradation of the rest of the orbit.

TABLE 5

	CASE 1 - DISTANCE		CASE 4 - ANGULAR MEASUREMENTS	
	Noise + bias	Stations displaced	Noise + bias	Stations displaced
a	35 m	27 m	89 m	25 m
e	0.13×10^{-5}	0.99×10^{-6}	0.13×10^{-4}	0.36×10^{-6}
i	0.154×10^{-4} rad	0.117×10^{-4} rad	0.41×10^{-4} rad	011×10^{-5} rad

6. Deviations in the orbital parameters

In Table 6 is shown an example of the differences between the nominal orbital parameters obtained from "perfect" measurements, and those obtained from measurements containing errors.

TABLE 6

	CASE 1		CASE 2	
	Noise + bias	Stations displaced	Noise + bias	Stations displaced
$\frac{\Delta a}{a}$	7 m	9 m	14 m	0
Δe	1.3×10^{-6}	$7. \times 10^{-9}$	$1. \times 10^{-7}$	$8. \times 10^{-8}$
Δi	$8. \times 10^{-4}$ deg	$4. \times 10^{-6}$ deg	$3. \times 10^{-4}$ deg	$3. \times 10^{-6}$ deg

It should not be forgotten that Case 4 deals with angular measurements, and consequently with two measurements. There is thus a greater difference than in Case 1, which deals only with measurement of distance.

V. Remarks

1. The curves of Figures 9, 10, and 11 show the change with time of errors in longitude, latitude, and altitude for several cases.

2. We shall summarize here the results published in an article of Dr. Chreston and F. Martin: "Accuracy of Satellite Orbits Obtainable by Synchronous Satellite Tracking".

The simultaneous measurements were of distance and radial velocity. They assumed one observation every minute continuously for a period of two weeks.

The stationary satellite was located over the Atlantic and was tracked by three stations: Winkfield, Pretoria, and Merritt Island: they thus had a good tracking geometry.

The sources of error considered were:

- 15 meters in the three components of the stations;
- noise of 1 meter in the distance;
- noise of 0.2 cm/sec in the velocity;
- bias of 5 meters in the distance.

The minimum precision with which the position of the satellite is restored is given below:

$\sigma_{latitude}$: 5 meters
$\sigma_{longitude}$: 125 meters
$\sigma_{altitude}$: 6 meters

The authors draw three conclusions from the study:

- The longitude component is the most poorly restored. The relationship of the errors corresponds in general to what we have found.

- The effect of a bias of 5 meters in the measurements is of the same order of magnitude as that of an error of 15 meters in the position of the stations. We likewise find equivalent orders of magnitude. /20

- The errors due to the position of the stations and to systematic errors of measurement predominate. The contributions of noise and potential model are negligible. The frequency and number of measurements can reduce this contribution to the noise, and a better-developed model can be used if necessary.

3. "Computer time"

During this study we have never considered a "computer time" restriction. A first estimate can, however, be made.

Geometric localization with the aid, for example, of three distance measurements from three stations made in a highly regular manner involves no significant computer time, and requires no extrapolation to have the position of the satellite.

By using a differential correction program, the CDC 6600 computer would need about a quarter-hour per day to obtain a set of orbital parameters, and a dozen minutes to dispose of the position of the satellite in increments of one minute for a period of 24 hours. On the other hand, if failure of a station should cause us to lack measurements, the orbital parameters would allow the position of the satellite to be deduced with an accuracy, so to speak, equivalent to that achieved during routine tracking.

VI. Conclusion

In localization of a stationary satellite, the longitude is the most difficult component to determine: one can say that in general there is a factor of ten between the restoration of the latitude and that of the longitude.

Use of distance measurements (with noise of 100 m + bias of 100 m + uncertainty of 50 m in the position of the stations) gives knowledge of the /21
longitude to about 8 km. The same order of magnitude is obtained by angular measurements (noise of 2×10^{-4} + bias of 2×10^{-4} in the cosines of the angles), since there are in fact two measurements, and thus a greater dispersion.

Simultaneous use of these two methods of measurement allows localization to be improved by a factor of about two.

The errors in the position of the stations must be taken into consideration. Their effects can be reduced by a factor of two to five by special determination of the positions.

It is indispensable that the observations be well-distributed in time to assure a good coverage of the orbit.

GENERALIZATIONS

In order to obtain an image which would require no processing to be used, one needs an orbit with eccentricity of 3×10^{-5} and inclination of 8×10^{-3} degrees. Supposing that such an orbit could really be obtained, and ignoring the number of corrections required to maintain the position, in order to be able to make indispensable trajectory corrections, it would be necessary to estimate the orbital elements to at least 10%, which gives:

$$e = 3 \times 10^{-6} \qquad \text{and} \qquad i = 8 \times 10^{-4} \text{ degree}$$

Use, for example, of a distance measurement from a single station, containing a bias of 100 m permits such a level to be achieved if all other sources of error are neglected. In particular, one would have to reduce to a minimum, the error caused by lack of precision in the position of the stations. It is possible to know a station to about 5 meters by using the results of geodetic studies in progress or planned.

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